See the entire SIAM Guidebook for Mathematical Modeling in the course website.

7. PUTTING IT ALL TOGETHER

Now that a model has been created, solved, and assessed, the time comes to write everything up as a polished solution paper. This step is just as important as the effort necessary to get to this point. Keep in mind that you are the expert about the problem, and now your role is to explain what you did in detail to people unfamiliar with your solution approach. To this end, it is critical that you take good notes from the initial brainstorming process through the final analysis to be sure you have kept track of all the assumptions you made. Good writing also takes time, so be sure to allocate a period of time to step back from the math modeling and focus on quality writing. In this section we discuss how to structure your report and some key points for successful technical writing.

This step is just as important as the effort necessary to get to this point. Keep in mind that you are the expert about the problem.

STRUCTURE

A technical report typically starts with a summary page, also called an *executive summary* or an *abstract,* that is of one page or shorter. This is not an introduction; it's actually a place to summarize how the problem was solved and to provide a brief description of the results. It might seem strange to put the conclusion at the beginning, but this "bottom-line up front" approach is convenient for those reading your report.

The abstract or summary page should restate the problem, briefly describe the chosen solution methods, and provide the final results and conclusions. You should describe your results in complete sentences that can stand on its own, without using variables. The summary lets the reader know what to expect in the report but does not overwhelm him or her with unfamiliar mathematical notation. Imagine that a reader will decide whether to continue reading the rest of your paper itself based on this abstract. As an example, see the solution to the recycling problem.

After the summary, the paper should include a formal introduction that includes a restatement of the underlying real-world application as if the reader does not have any prior knowledge. This section usually contains some motivation or relevant background information as well but should not include a lengthy history lesson. Both the general modeling question as well as the concise problem statement that you developed should be at the forefront of this section. This section must provide a paragraph that describes how you approached the problem. For example, if we consider the task of ranking roller coasters based on how thrilling they are, then it would help to define "thrilling" up front. For example, "Our model is based on the notion that rides with high accelerations, inversions, and significant heights are thrilling." Note that this statement doesn't exactly explain how those features are quantified or implemented for a mathematical ranking system for thrilling roller coasters, but it does give the reader an idea of what will be showing up later in the document. It may seem counterintuitive, but the introduction and abstract are typically written last. This is because, after all else has been written, the author has a complete picture of the manuscript and may then best tailor these sections accordingly.

The body of the solution paper will likely be several pages long and split up into sections about assumptions, variables, the model, the solution process, analysis, and overall conclusions. Let the reader know about the overarching assumptions you made to make the problem solvable. Some specific assumptions may need to be included again later, within the paper's main text, in order to clarify certain ideas as they are developed. However, the most important message here is that all assumptions are included and listed at some point in your write-up. You should be sure to justify why those assumptions are reasonable and include citations as needed. Plagiarism of any kind is never acceptable.

When you next begin to describe your model and how you solved it, clearly state the variables you will be using and the corresponding units. If there are relationships between variables, explain where the come from and, if needed, refer to any necessary assumptions. Mathematical equations and formulas should be centered, each occuring on their own line. We provide some more specific details on this in the following section.

Finally, the paper must have a conclusion section that recaps the important features of the model. It is critical that this section includes an analysis of your results, as described in the previous section. An honest assessment of the strengths and weaknesses is important. In particular, you can comment on how the model can be verified and how sensitive the model is to the assumptions. We proceed by giving some tips about technical writing.

TECHNICAL WRITING DOS & DON'TS

1. DO NOT WRITE A BOOK REPORT.

It is critical that the narrative of your solution doesn't read like a story about how you came up with your model. For example, consider the following two excerpts about an assumption made for the recycling model.

Example 1:
A study of drop-off recycling participation in Ohio found that 15.5% of citizens who do not have access to curbside recycling use drop-off recycling [8]. We have assumed that this data is representative of the U.S.

Example 2:
We were stuck because we did not know how many people in the U.S. recycle. We googled and found an article that Ohio's participation in drop-off recycling was 15.5% for people who did not have access to curbside recycling, so we used that number in our model.

The second example is written in a way that makes the assumption sound invalid or that it was chosen only because no other information could be found. However, the first one sounds as though some research was done and a useful and legitimate source was identified, which provided an applicable statistic.

2. DO NOT USE WORDS WHEN USING MATHEMATICS WOULD BE MORE APPROPRIATE.

Which of the following is more effective?

Example 1:
For an ideal gas, we have

$$P = \frac{nRT}{V} ,$$

where P is the absolute pressure of the gas, V is the volume of the gas, n is the amount of substance of gas (measured in moles), T is the absolute temperature of the gas, and R is the ideal, or universal, gas constant.

Example 2:
For an ideal gas, the absolute pressure is directly proportional to the product of the number of moles of the gas and the absolute temperature of the gas and inversely proportional to the volume of the gas, with proportionality constant R, the ideal, or universal, gas constant.

In this case, the mathematics is easier to follow, and you can imagine that the more complicated the calculations, the harder it would be to try to describe in words.

3. DO USE PROPER SENTENCE STRUCTURE WHEN EXPLAINING MATHEMATICS.

Communicating mathematics requires proper punctuation, such as periods at the end of a computation if the computation ends the sentence, as in Example 1 below. Use commas when appropriate.

Example 1:
If s is the length of the side of the square box, then the area of a side is given by

$A = s^2$,

and the volume is given by

$V = s^3$.

Example 2:
If s is the length of the side of a square box then we can find the area and volume.

$A = s^2$

$V = s^3$

4. DO NOT SUBSTITUTE MATHEMATICAL SYMBOLS FOR WORDS WITHIN SENTENCES, AS IN THE SECOND OF THE FOLLOWING TWO EXAMPLES.

Example 1:
For this work L is the length of the side of a rectangle.

Example 2:
For this work L = the length of the side of the rectangle.

5. DO PAY ATTENTION TO SIGNIFICANT FIGURES.

For example, your calculator might read a value of 27.3416927482, but you may not need to report all of those digits unless you are trying to show accuracy in the later decimal places.

6. DO USE SCIENTIFIC NOTATION WHEN NUMBERS VARY BY ORDERS OF MAGNITUDE,

meaning that the exponent is really what matters in understanding the significance of the value. For example, the diameter of the sun is $1.391e^{6}$ km, while the diameter of a baseball is $2.290e^{-4}$ km.

7. DO LABEL FIGURES

and use a large enough font so that the axes are clearly readable.

8. DO NOT FORGET TO INCLUDE UNITS AS APPROPRIATE.

9. DO CHECK CAREFULLY FOR SPELLING AND GRAMMAR MISTAKES,

especially those that spell check might miss. For example, it's easy to confuse *their*, *there*, and *they're*.

10. DO GIVE CREDIT WHERE CREDIT IS DUE.

This means including citations and building your bibliography as you go.

Technical writing takes practice, but the end result should be something of which to be extremely proud. In reviewing your final paper, you can step back and look at all you have accomplished throughout the modeling process. Ultimately your model can lead to the creation of new knowledge and provide deeper insight about the world we live in. Remember that modeling also takes practice so the next time you tackle an open-ended problem you will already have a new set of tools that will make the entire experience go more smoothly.

IN SUMMARY

1. TAKE NOTES THROUGHOUT YOUR ENTIRE MODELING PROCESS SO THAT YOU DO NOT LEAVE OUT ANYTHING IMPORTANT, ESPECIALLY ASSUMPTIONS MADE ALONG THE WAY.
2. GIVE YOURSELF ENOUGH TIME TO FOCUS ON THE WRITING PROCESS AND TO PROOFREAD THE REPORT.
3. KEEP IN MIND THAT THIS IS A TECHNICAL DOCUMENT, NOT A STORY ABOUT YOUR MODELING EXPERIENCE.
4. FOLLOW THE GUIDELINES FOR TECHNICAL WRITING.
5. SOME ADDITIONAL REFERENCES ON TECHNICAL WRITING CAN BE FOUND AT [3].
6. PAT YOURSELF ON THE BACK FOR YOUR ACCOMPLISHMENTS.

B. THE 2013 M^3 CHALLENGE PROBLEM AND THE SOLUTION PAPER FROM TEAM 1356

Waste Not, Want Not: Putting Recyclables in Their Place

Plastics are embedded in a myriad of modern-day products, from pens, cell phones, and storage containers to car parts, artificial limbs, and medical instruments; unfortunately, there are long-term costs associated with these advances. Plastics do not biodegrade easily. There is a region of the Northern Pacific Ocean, estimated to be roughly the size of Texas, where plastics collect to form an island and cause serious environmental impact. While this is an international problem, in the U.S. we also worry about plastics that end up in landfills and may stay there for hundreds of years. To gain some perspective on the severity of the problem, the first plastic bottle was introduced in 1975 and now, according to some sources, roughly 50 million plastic water bottles end up in U.S. landfills *every day*.

The United States Environmental Protection Agency (EPA) has asked your team to use mathematical modeling to investigate this problem.

How big is the problem? Create a model for the amount of plastic that ends up in landfills in the United States. Predict the production rate of plastic waste over time and predict the amount of plastic waste present in landfills 10 years from today.

Making the right choice on a local scale. Plastics aren't the only problem. So many of the materials we dispose of can be recycled. Develop a mathematical model that a city can use to determine which recycling methods it should adopt. You may consider, but are not limited to:

- providing locations where one can drop off pre-sorted recyclables
- providing single-stream curbside recycling
- providing single-stream curbside recycling in addition to having residents pay for each container of garbage collected

Your model should be developed independent of current recycling practices in the city and should include some information about the city of interest and some information about the recycling method. Demonstrate how your model works by applying it to each of the following cities: Fargo, North Dakota; Price, Utah; Wichita, Kansas.

How does this extend to the national scale? Now that you have applied your model to cities of varying sizes and geographic locations, consider ways that your model can inform the EPA about the feasibility of recycling guidelines and/or standards to govern all states and townships in the U.S. What recommendations does your model support? Cite any data used to support your conclusions.

Submit your findings in the form of a report for the EPA.

The following references may help you get started:

http://www.epa.gov/epawaste/nonhaz/municipal/index.htm
http://5gyres.org/what_is_the_issue/the_problem/

Moody's Mega Math Challenge supports Mathematics of Planet Earth (MPE2013).
www.mpe2013.org

Organized by SIAM Society for Industrial and Applied Mathematics

Funded by The Moody's Foundation

Analysis of Plastic Waste Production and Recycling Methods

EXECUTIVE SUMMARY

In 2010 alone, the U.S. generated approximately 250 million tons of trash [1]. Much of this waste consisted of plastics, which build up in landfills and flow into oceans through storm drains and watersheds [2], breaking up into little pieces and absorbing contaminants in the process. A major method to reduce waste is recycling, where materials like glass, paper, and plastic are reformed to create new products. There are many different methods of collection of recyclable materials, including drop-off centers, where citizens transport their recyclables; single stream curbside collection, where the city collects the recyclables of each household; and dual stream curbside collection, where the city collects recyclables that are pre-sorted by each household. To encourage or subsidize recycling programs, some cities may implement a Pay-As-You-Throw (PAYT) program, where citizens pay a fee based on the amount of garbage they throw away.

The EPA tasked us to analyze the production and discard rate of plastic waste over time. We were also asked to create a model of possible methods for recycling collection to determine which methods are appropriate for what cities. Using a linear regression model over years passed since 2000, we estimated that 35.1 million tons of plastic waste will be discarded in 2023. We also modeled the use of drop-off centers, single stream curbside collection, and dual stream curbside collection to calculate the total amount of recyclables collected as well as the cost to the city using each recycling method.

For collection using drop-off centers, we developed a simulation that randomly simulated the number of households who would recycle when drop-off centers were placed around the city. The simulation took into account the area, population, average household, maximum distance citizens are willing to travel, and number of drop-off centers. Using these data, we calculated the amount recycled and then calculated the net cost to the city by subtracting operating costs of Material Recovery Facilities (MRFs) from revenue generated by selling recycled products.

For curbside collection, we calculated the number of trucks needed to service a given city, based on population density. Based on labor, upkeep, and fuel, we calculated the costs of a curbside collection program. Again, using the calculated amount of collected material, we determined the net revenue generated by these products.

We determined that by using a drop-off center method, Fargo and Wichita would generate profits, while Price would incur costs that could be partially covered by using Pay-As-You-Throw. Using a curbside collection method, Fargo and Price would incur costs that could be partially covered by Pay-As-You-Throw, while Wichita would generate profits using either single or dual stream collection. Thus, either drop-off or curbside collection methods may be feasibly implemented in cities around the U.S., depending on population and area of each city. We concluded that small cities tend to incur net costs from recycling programs, while larger cities like Wichita may profit from using a dual stream curbside collection program.

To assess use of recycling programs on a national level, we programmed a computer simulation generating an image of all the counties of the U.S., where blue dots on the U.S. map represented counties where at least one of our three proposed recycling programs earned a net profit. In general, we recommend that the EPA extend recycling program guidelines to the national level.

I. INTRODUCTION

1. Background

Each year, the U.S. consumes billions of bags and bottles. However, of the plastics that the U.S. produces, only 5% is recovered [2]. Unrecycled plastics present a growing hazard because they contain dangerous chemicals like polycarbonate, polystyrene, PETE, LDPE, HDPE, and polypropylene, which accumulate over time and build up in our oceans and landfills. As such, it is important to assess the scale of our plastic waste production problem over time.

Our foremost method of reducing wastes like plastics is through recycling, where useful materials including glass, plastic, paper, and metals are recovered so that they may be used to create new products [3]. There exist several methods of recycling collection; in general, cities may use either use drop-off centers or curbside collection. With drop-off centers, the residents carry the burden of transporting their recyclable waste, while curbside collection places this burden on the city. If a city implements curbside collection, it may choose to use single stream, dual stream, or pre-sorted methods; in single stream, all recyclables are collected as one unit, whereas in dual stream, recyclables are separated into paper and glass, cans, and plastic [4]. Further separation exists with the pre-sorted collection method, where recyclables are fully separated by material type [5]. There are advantages and disadvantages associated with each method of collection, and in choosing the type of recycling program to implement, cities must consider, among other factors, the practicality of individual household collection, as well as the volume of recyclables that would be collected using each program [6]. Some communities may use Pay-As-You-Throw (PAYT) programs, which encourage residents to recycle their waste so as to avoid fees dependent on the weight of their trash [7]. We assess in this analysis whether it is more efficient to use drop-off centers or curbside collection, depending on the city where the recycling program is being implemented, as well as the effect of using PAYT programs to generate additional revenue for the city.

2. Restatement of the Problem

In this analysis, we were requested by the EPA to create a model to predict the change in plastic production rate over time, as well as the amount of plastic waste in landfills in the year 2023. We were further asked to look at various recycling methods, not limited to the recycling of plastics, and to analyze the recycling method a city should develop, using as sample points the cities of Fargo, North Dakota; Price, Utah; and Wichita, Kansas. Finally, the EPA requested that we provide recommendations for developing recycling methods on the national level based on the model we designed.

3. Global Assumptions

Throughout our analysis, we will make the following assumptions:

1. A city's population is approximately evenly distributed. Population mostly varies on a large scale: in the small microcosm of a city, the population density will not vary much.
2. A city's shape is approximately square. Most cities are shaped like this, as are the three sample cities we were provided with.
3. A city's roads are laid out in a grid plan. The popularity of the grid plan is pervasive, dating back to Ancient Rome, and most cities are organized as such, like our three sample cities. [8, 9, 10].

4 A household's recycling stance is consistent. That is, a household that recycles always recycles, and a household that does not will never recycle. Recycling is a habit, and households that recycle tend to recycle consistently.

II. ANALYSIS OF THE PROBLEM AND THE MODEL

1. Plastic Waste Production

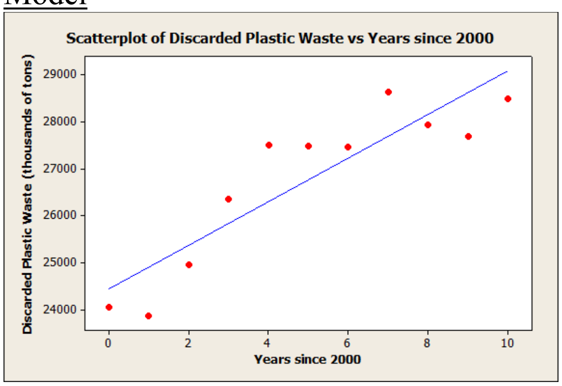
Assumptions

1 We used data collected from the past ten years because the first plastic bottle was introduced in 1975 [11], and recycling has only become important recently. In other words, values used before 2000 would not adequately take into account the recycling methods which have now become widespread.

Approach

We created our model by performing linear and logistic regressions on the amount of plastic waste discarded per year for the last decade in thousands of tons as provided by the EPA [1].

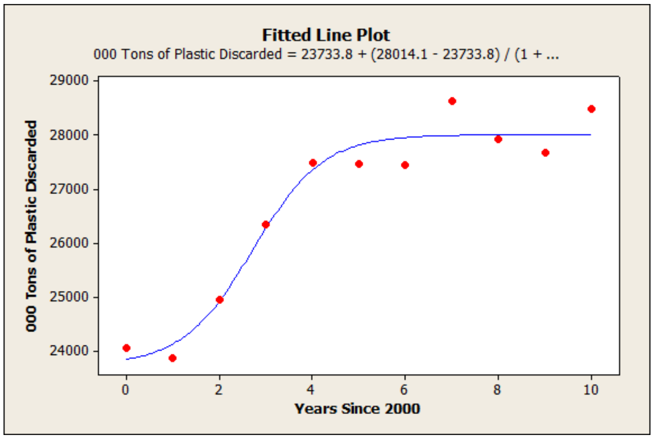
Model



*Discarded Plastic Waste (thousands of tons) = 463.27 * (years since 2000) + 24443.6*
R^2 = .803; S = 801

The R^2 value of .803 means that 80.3% of the variability in the amount of plastic waste discarded is explained by the linear relationship between years passed since 2000 and plastic waste amount. The standard deviation of the residuals was 801.

Based on this model, the amount of unrecycled plastic waste discarded in 2023 will be 463.27 * 23 + 24443.6 = 35098.81 thousands of tons, or 35.1 million tons.



Discarded Plastic Waste (thousands of tons) = 23733.8 + (28014.1 - 23733.8) / (1 + exp((Years since 2000 - 2.71325) / -0.752611))
S = 422

The previously mentioned R^2 value only makes sense under the assumption that the linear model was appropriate. Since there is a prominent bend in the data, we fit them with a logistic curve as well. A statistical software found the four parameters using a successive approximation method, and produced the model above.

The standard deviation of the residuals in this model is only 422, which is almost twice as small as it was in the linear model. Unfortunately, this model assumes that the tonnage of discarded plastic waste will level off, which is not entirely reasonable. It does, however, give a best-case result (e.g. if recycling initiatives work perfectly). The projected value of discarded waste for 2023 is 28014.1 thousand tons (within 4 decimal places), which is the maximal value according to the model.

In summary, the linear model (which seems to overpredict the later values) yields a value of 35.1 million tons, while the logistic model (which levels off) predicts that it will level off at 28.0 million tons. The US population has been increasing linearly since 2000 [12], so the linear model gives a more plausible value for the next ten years.

2. Recycling Methods

Assumptions

1. City shape can be approximated as a square or diamond. Most cities in the U.S. are square-like in shape, including Fargo, North Dakota, Price, Utah, and Wichita, Kansas.
2. The streets of the city are laid out in a grid. Many large cities have streets following a grid, including Fargo, North Dakota; Price, Utah; and Wichita, Kansas all use grid systems
3. There is no overlap between use of drop-off centers and curbside collection.
4. The composition of recyclables in the MSW stream is fixed over the entire planning horizon.

Model 1: Drop-off Centers

Approach

Assumptions

1. Each household makes a collective decision on whether or not to recycle because it is convenient for a household to transport all of their recyclables to a drop-off center together.
2. The probability of a household's deciding to recycle varies linearly with the household's distance to the nearest drop-off center.
3. Recycling households recycle all recyclable waste.

To assess the amount of recyclables collected by a recycling program dependent on drop-off centers, we created a computer simulation where we assumed uniform population density and where we place equally spread drop-off centers around the city, as many as would fit without overlapping coverage. To determine whether each household would recycle, a random number from 0 to 1 is generated, and if the number is less than the household's probability of recycling, which we assumed varies linearly with the household's distance to the nearest drop-off center, the household recycles. We also determined the cost of maintaining each recycling center and the revenue the center would generate, and used these data to calculate the total cost to the city of the drop-off center program. In our simulation, we accepted as inputs the area of the city, population of the city, average number of people in a household, maximum distance citizens are willing to travel, and number of drop-off centers.

Taxicab Distance

Because streets are assumed to be organized in a grid, we calculate distance as "taxicab distance", or distance in which the only path allowed consists of horizontal and vertical lines. In other words, given p_x and p_y as the coordinates of the drop-off center, and x and y as the coordinates of the household, the distance between them, d, can be calculated as:

$$d = |y - p_y| + |x - p_x|$$

A Household's Maximum Distance to a Drop-off Center

<u>Assumption</u>

1. Recycling households make biweekly trips to a drop-off center.

We recommend that cities conduct a survey to determine the distance their citizens are willing to travel in order to recycle, though we calculated this distance in our model. U.S. citizens are willing for their household to pay $2.29 a month for curbside collection [13]. Since this is the amount that they are willing to pay to recycle at greatest convenience, we can assume that it is equivalent to the maximum amount they are willing to pay as the driving cost to a drop-off center.

The average price of a gallon of gas is $3.784 [14] and the average mileage of a passenger car in 2010 was 23.8 mpg [15]. The cost of traveling a distance d is:

$$Cost = (\$3.784/gallon) * d / (23.8\ miles/gallon)$$

The distance citizens are willing to travel each week is:

*($2.29 dollars/month) / (4.35 weeks/month) = $0.53 dollars/week = ($3.784/gallon) * (d miles/week) / (23.8 miles/gallon)*
*d miles/week = ($0.53 dollars/week) * (23.8 miles/gallon) / ($3.784 dollars/gallon) = 3.33 miles/week*

Since citizens must drive to the drop-off center and back, the maximum distance driven to the drop-off center is 1.665 miles/week. Assuming that households make biweekly recycling trips, the maximum distance from a household to a drop-off center for the household to consider recycling is 1.665 miles/week * 2 weeks = 3.33 miles.

A study of drop-off recycling participation in Ohio supports our model, finding that the functional usage area of a full-time urban drop-off center is about 3.5 miles [16].

Number of Recycling Households Covered by a Drop-off Center
<u>Assumption</u>

1. The available data from Ohio are representative of the U.S. as a whole.

Each drop-off center will receive recyclables from households up to 3.33 miles away. Using taxicab distance, which only allows horizontal and vertical movement, the area within 3.33 miles is bounded by a diamond (a square rotated 45°). The diagonal of the diamond is twice the distance from the center to a corner, or 2 * 3.33 miles = 6.66 miles. Since the diamond is a square, diagonal length = square root(2) * side length, so the side length is 4.71 miles. The area of the diamond is side length ^ 2 = 22.18 sq. mi. This is the coverage area of the drop-off center, which contains all the households that will consider using the drop-off center.

The number of households in the drop-off center's coverage area is:

*Households = 22.18 sq. mi * (population / land area) / (average household size)*

A study of drop-off recycling participation in Ohio found that 15.5% of citizens who do not have access to curbside recycling use drop-off recycling [16]. Assuming that this data is representative of the U.S. as a whole, the number of recycling households covered by each drop-off center is:

*Recycling households = 22.18 sq. mi * (population / land area) / (average household size) * .155*

In our simulation, we assigned 15.5% as the median household probability of recycling. The closer a household is to the drop-off center, the more likely it is to recycle. Within the drop-off center coverage area, the closer half of households has a greater than 15.5% recycling probability and the farther away half of households has a less than 15.5% recycling probability. The distance from the center to the boundary of the closer half of households is the diagonal of the square with half the area of the entire coverage area, which is:

*Halfway distance = square root(22.18 sq. mi. / 2) * square root(2) = 4.71 miles*

We assumed that the probability of a household recycling varies linearly with the distance to the nearest recycling center. At a distance of 4.71 miles, the probability is 15.5%. At the boundary distance of 6.66 miles, the probability is 0%. Extending the line through these points, at the center, with a distance of 0 miles, the probability is 52.9%. In our simulation, the number of recycling households covered by each drop-off center is approximately the same as that calculated using the formula previously given.

Drop-off Center Placement

In our simulation, we placed as many drop-off centers as possible in each city so that none of the coverage areas overlap, with at least one drop-off center in each city. The cost efficiency of drop-off centers decreases when their coverage areas overlap.

Annual Amount Recycled

The average American generates 4.5 pounds of waste per day [17], about 75% of which is recyclable [18]. Thus, the average American generates 4.5 pounds * 0.75 = 3.375 pounds of recyclable waste per day.

*Annual Amount Recycled (tons) = (recycling households) * (average household size) * 3.375 lb * 365 days/year * 0.005 lb/ton * (# drop-off centers)*

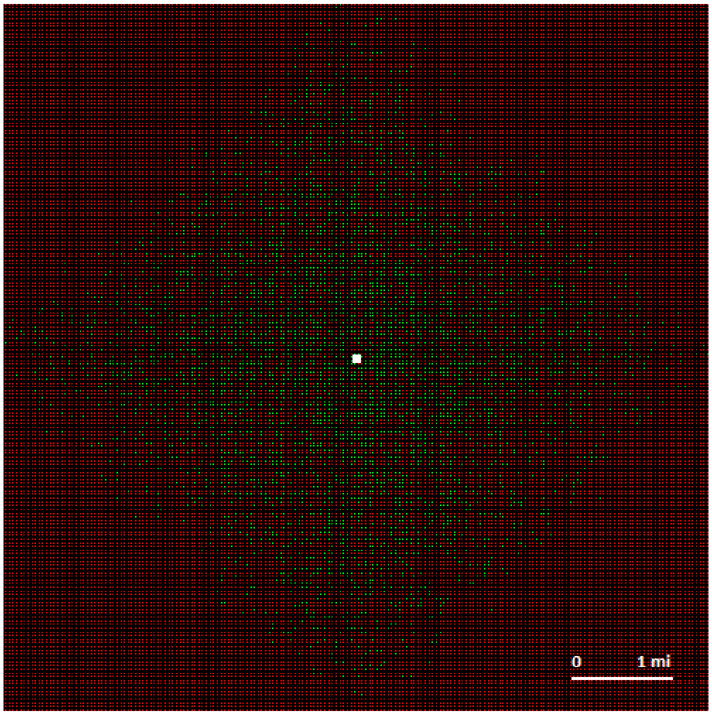
This formula can be used in place of our simulation to calculate annual amount recycled, as long as there is no overlap between drop-off center coverage areas and the drop-off center coverage area is entirely contained within the city. For example, because the drop-off center coverage area (22.18 sq. mi.) is much larger than the area of Price, Utah (4.2 sq. mi.), this formula cannot be used in place of our simulation for Price, Utah.

Using our simulation, we were able to calculate the annual amount recycled for Fargo, North Dakota; Price, Utah; and Wichita, Kansas, as well as to visualize the households contributing recyclables to each city. In the screenshots below, the white dots represent the drop-off centers; the green dots represent households that are recycling; and the red dots represent households that are not recycling.

Fargo, North Dakota

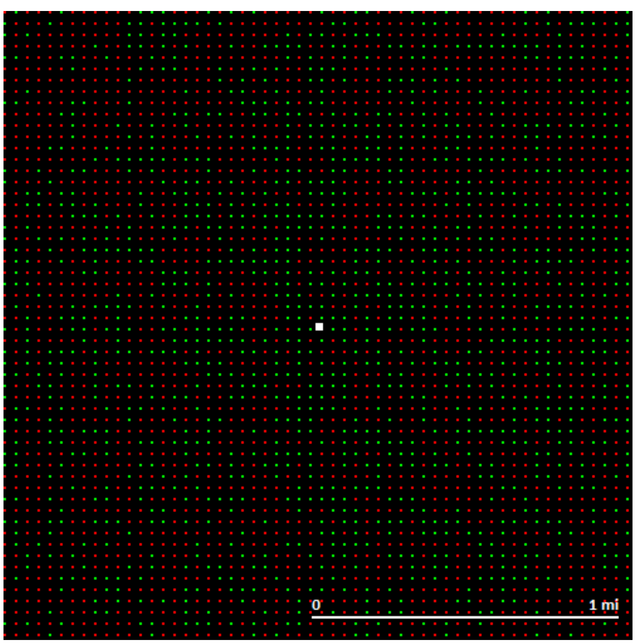
Annual Amount Recycled (tons) = 5209.66

Number of people recycling= 8458



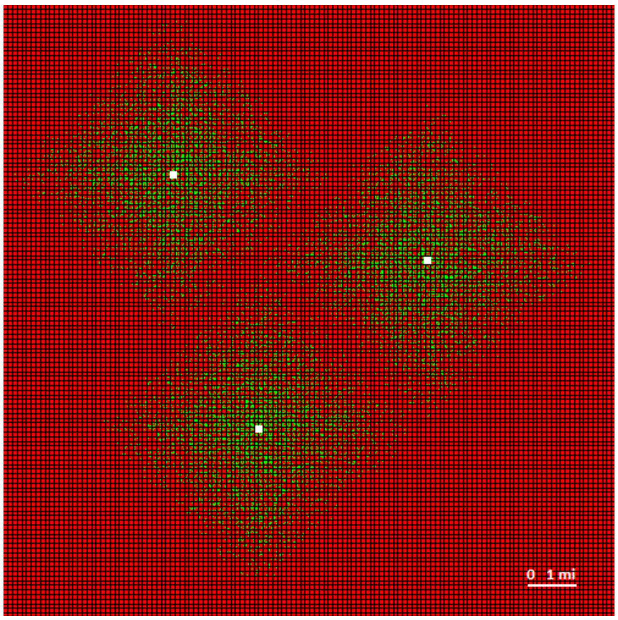
Land Area = 48.82 sq. mi.
Population = 105,549 people
Average household size = 2.15 people

Price, Utah
Annual Amount Recycled (tons) = 1876.88
Number of people who recycle = 3047



Land Area = 4.2 sq. mi.
Population = 8,402 people
Average household size = 2.60 people

Wichita, Kansas
Annual Amount Recycled (tons) = 16929.56
Number of people who recycle = 27486



Land Area = 159.29 sq. mi.
Population = 382,368 people
Average household size = 2.48 people

Drop-off Center Cost

A report by design engineering company R.W. Beck, Inc. recommends front load dumpsters as the most cost-effective type of drop-off center. Under this plan, front load dumpsters would be set up at each drop-off center site and recyclables would be collected in two streams, commingled containers and paper. The annual cost of a front load dumpster site is about $5,575 per year [19]. Thus, the total annual cost of drop-off centers is:

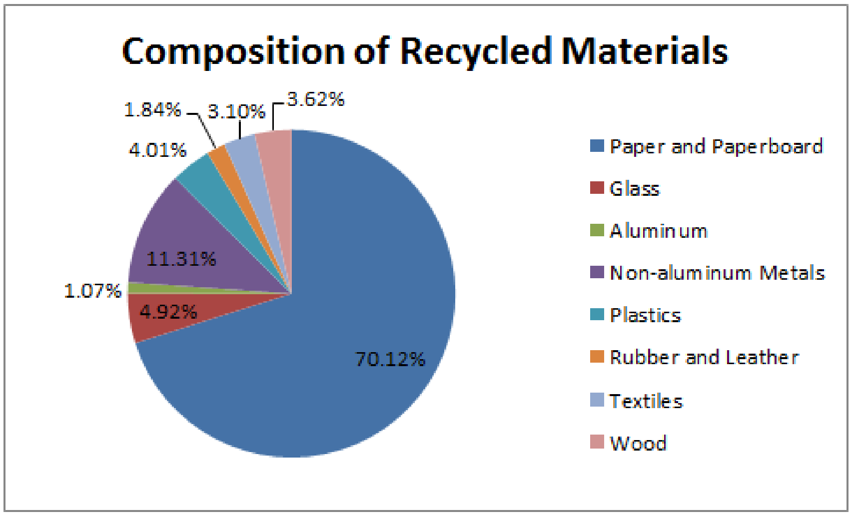
*Annual cost of drop-off centers = $5,575 * (# drop-off centers)*

Revenue Generated

To calculate the total revenue per ton generated from selling recycled products, we used the following formula, taking into account the market price per ton for each product [20, 21, 22, 23, 24, 25, 26, 27]:

$Revenue\ per\ ton = Revenue_{Paper} + Revenue_{Glass} + Revenue_{Ferrous\ Metals} + Revenue_{Aluminum} + Revenue_{Plastic} + Revenue_{Textiles} + Revenue_{Wood} = (.7012*\$112.82) + (.0492*\$13) +$

*(.1131*$217.75) + (.0107*$310) + (.0401*$370) + (.031*$100) + (.0362*$296) +($135*.0180) = $128.78 per ton of recycled material*



[1]

Based on a study conducted on recycling collection and processing options in New Hampshire [28], cities can decide between small, medium, and large Materials Recovery Facilities (MRFs) depending on the annual tonnage. The cost per ton using dual stream and cost per ton using single stream varies depending on the size of the MRF. For drop-off centers, we are assuming that dual stream is used.

Fargo, North Dakota

We calculated that Fargo would collect 5,209.66 tons of recyclables. This suggests that a medium tonnage mini MRF, which has an annual tonnage of 5,283, is sufficient for the city. The cost per ton of a medium mini MRF using dual stream is $124.62. Since the material revenue per ton was previously found to be $128.78, we can calculate the net cost per ton as:

Net cost = $124.62 - $128.78 = -$4.16

The total cost to the city can then be calculated as:

*Total cost = -$4.16 per ton * 5,209.66 tons + $5,575 per drop-off container * 1 container = -$16,097.19 (profit)*

Price, Utah

We calculated that Price would collect 1876.88 tons of recyclables. Price would use a low tonnage mini MRF, and the net cost per ton would also be $89.69. Then, the total cost to the city is:

*Total cost = $89.69 per ton * 1876.88 tons + $5,575 per drop-off container * 1 container = $173,912.37*

Wichita, Kansas

We calculated that Wichita would collect 16,929.56 tons of recyclables, suggesting that Wichita would require a high tonnage mini MRF, which has an annual tonnage of around 7,500. For a high tonnage mini MRF, the cost per ton for dual stream is $95.40. Since the material revenue is $128.78 per ton, the net cost per ton is:

$$\text{Net cost} = \$95.40 - 128.78 = -33.38$$

This represents a profit of $33.38 per ton of recycled material. The total cost to the city is then:

$$\text{Total cost} = -\$33.38 \text{ per ton} * 16{,}929.56 \text{ tons} + \$5{,}575 \text{ per drop-off container} * 1 \text{ container} = -\$559{,}533.71 \text{ (profit)}$$

Pay-As-You-Throw

If the city implements a Pay-As-You-Throw (PAYT) program, it will collect revenue from citizens who must pay an amount depending on the volume of waste they generate. We can calculate revenue generated by such a program by using the formula [29]:

$$Revenue_{PAYT} = (Weight_{Waste} / Volume_{Container} * Price_{Container} - Price_{Startup,\ Maintenance\ per\ day}) * Population$$

The average American generates 4.5 lbs of waste and recycles 1.5 lbs [17]. PAYT programs cost around $0.28 per capita, based on surveys of Wisconsin and Iowa [30]. We also simplified $\text{Volume}_{\text{Container}}$ * $\text{Price}_{\text{Container}}$ as Container price/pound, since the containers are meant to hold specific amounts of weight. Thus, the revenue generated by PAYT for citizens who recycle can be calculated as:

$$Revenue_{PAYT,\ Recycle} = ((4.5 \text{ lbs} - 3.375 \text{ lbs}) * \text{Container price/pound} - \$0.28/365) * Population_{Recycle}$$

$$Revenue_{PAYT,\ Don't\ recycle} = (4.5 \text{ lbs} * \text{Container price/pound} - \$0.28/365) * Population_{Don't\ recycle}$$

$$\text{Total revenue}_{PAYT} = Revenue_{PAYT,\ Recycle} + Revenue_{PAYT,\ Don't\ recycle} = ((4.5 \text{ lbs} - 3.375 \text{ lbs}) * \text{Container price/pound} - \$0.28/365) * Population_{Recycle} + (4.5 \text{ lbs} * \text{Container price/pound} - \$0.28/365) * Population_{Don't\ recycle}$$

Fargo, North Dakota

$$\text{Total revenue}_{PAYT} = (1.125 \text{ lbs} * x - \$0.28/365) * 8{,}458\ people_{Recycle} + (4.5 \text{ lbs} * x - \$0.28/365) * (105{,}549 - 8{,}458\ people_{Don't\ recycle})$$

Price, Utah

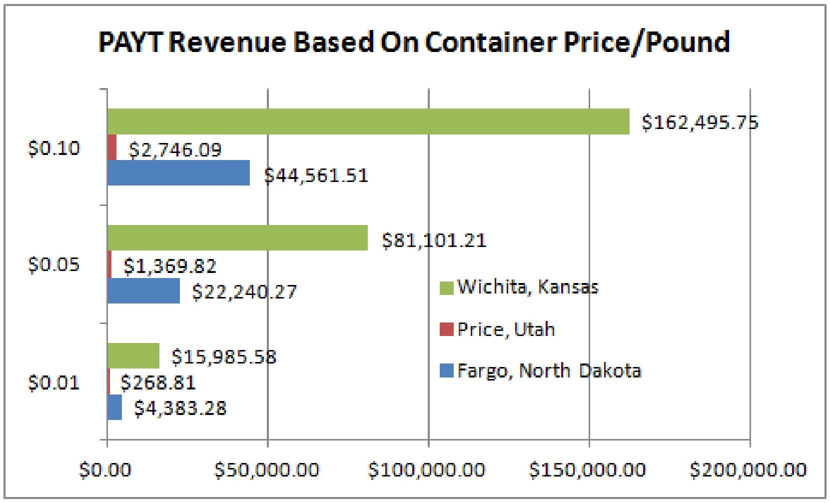
$$\text{Total revenue}_{PAYT} = (1.125 \text{ lbs} * x * - \$0.28/365) * 3047\ people_{Recycle} + (4.5 \text{ lbs} * x - \$0.28/365) * (8{,}402 - 3{,}047\ people_{Don't\ recycle})$$

Wichita, Kansas

$$\text{Total revenue}_{PAYT} = (1.125 \text{ lbs} * x - \$0.28/365) * 27{,}486\ people_{Recycle} + (4.5 \text{ lbs} * x - \$0.28/365) * (382{,}368 - 27{,}486\ people_{Don't\ recycle})$$

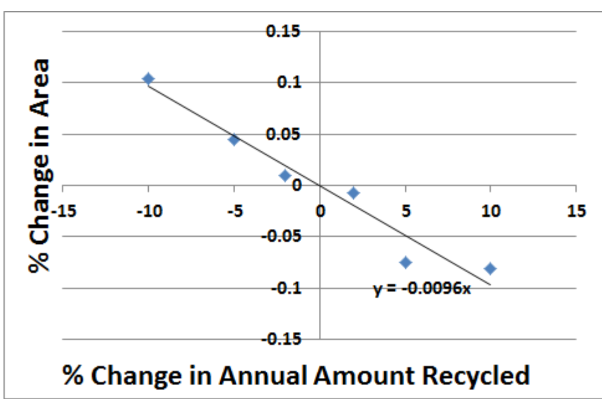
The following table provides the total revenue generated by a PAYT program if the container price per pound were $0.01, $0.05, or $0.10.

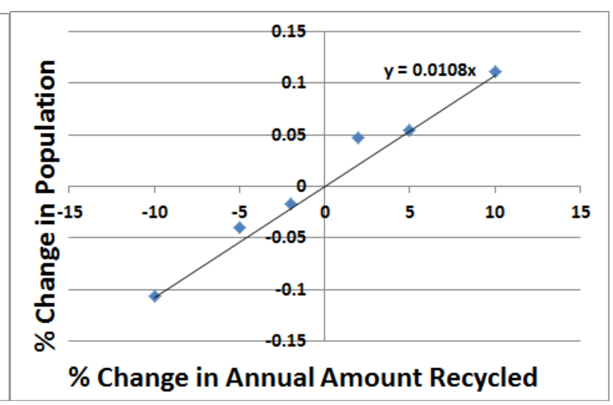
Container Price/Pound	Fargo, North Dakota	Price, Utah	Wichita, Kansas
$0.01	$4,383.28	$268.81	$15,985.58
$0.05	$22,240.27	$1,369.82	$81,101.21
$0.10	$44,561.51	$2,746.09	$162,495.75



Sensitivity Analysis

We tested the sensitivity of our simulation of the annual amount recycled in a city using a drop-off recycling program. We changed population and area by +/- 2%, 5%, and 10% and examined the resulting change in annual amount recycled. For simplicity, we only examined the changes for one of our sample cities: Fargo, North Dakota.





The annual amount recycled responds approximately linearly to both area and population. The response is not precisely linear because the randomness used in the simulation to determine whether each household recycles introduces some variation between different runs of the simulation. Since the slopes are small, a slight error in the initial parameters would not significantly change the simulation's output.

Model 2: Curbside Collection

Assumptions

1. Each city has only one recycling processing plant, located at the geographic center, as we found that one large-scale processing center is more than enough to cover one city's recycling needs.
2. Recycling collection comes biweekly.

Approach

We subdivided the city into zones for which one garbage truck was responsible. Each truck is responsible for driving to its zone, collecting all the recyclable waste it can, and delivering it to the central processing center, which then sorts and processes the recyclable waste.

Recyclable Waste Collected and Cost to City

We divide the cost to the city into three parts: the cost of gasoline, the wages of the truck drivers, and the price of truck upkeep. The cost is as follows:

*Cost = (Price of diesel fuel in dollars/gallon) * distance / (Truck miles/gallon) + (Num houses) / (Houses/hour) * (Driver wage/hour) + Truck_Upkeep*

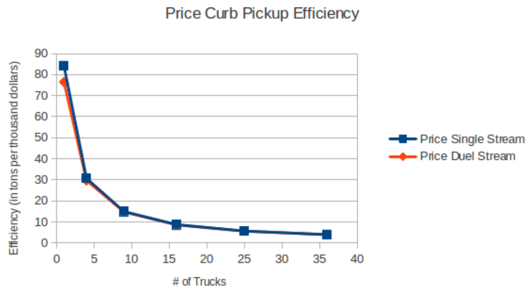
The number of houses visited per hour varies depending on whether a single stream or dual stream collection method is used; for single stream, 171 households are visited per hour, while for dual stream, 130 households are visited per hour [31]. The mileage of a truck is 5 mpg, with a cost of $4.02 per gallon. The average wage of a truck driver is $16 dollars/hour [19].

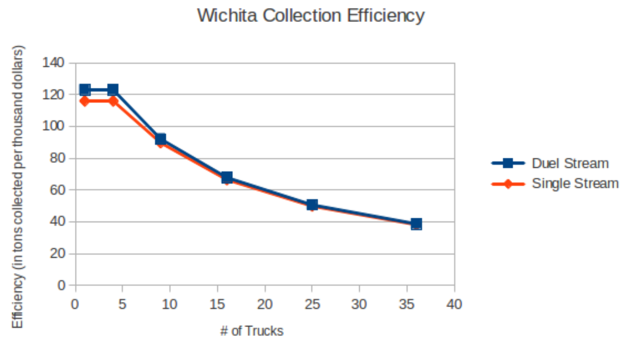
Thus, the formulas for single stream and dual stream collection costs are as follows:

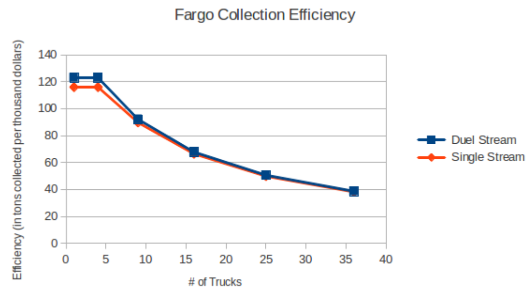
*Single stream cost = ($4.02 dollars/gallon) * distance / (5 miles/gallon) + (Num houses) / (171 houses/hour) * ($16 wage/hour)*

*Dual stream cost = ($4.02 dollars/gallon) * distance / (5 miles/gallon) + (Num houses) / (130 houses/hour) * ($16 wage/hour)*

We assume that a truck driver can only collect for 7 hours a day: (8 hour work day, minus an hour for lunch and driving). So, a truck driver has a maximum amount of households s/he can visit in a biweekly circuit (171 * 7 * 10=11970 for single-stream and 130*7*10=9100 for dual-stream). When a driver is tasked with more houses that s/he can visit, we simply used this ceiling. To demonstrate, the graphs below show efficiency, in terms of tons of recyclable waste collected per thousand dollars, versus the number of trucks in each city.







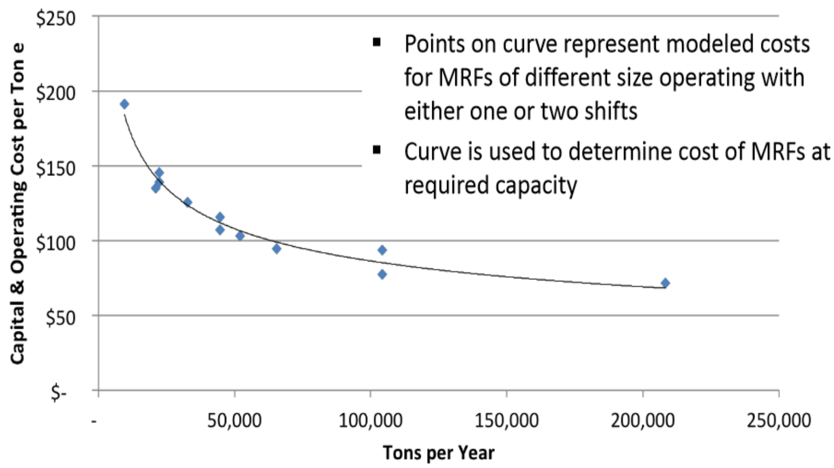
Using the model, we calculated the optimum number of trucks for each city for either dual stream or single stream curbside collection depending on the efficiency of the collection, quantified using the tons of recycled material collected per $1,000, and the total amount of recycled waste collected. The results for optimum number of trucks are shown below:

City	Single Stream	Dual Stream
Fargo	5	6
Price	1	1
Wichita	13	17

Given the optimum number of collection trucks, the annual cost and tons of recyclable waste collected can then be determined using our computer simulation.

City	Single Stream		Dual Stream	
	Tons of Waste	Collection Cost	Tons of Waste	Collection Cost
Fargo	25292.85	$205,787.20	25933.39	$319,383.30
Price	2064.37	$24,526.54	2064.37	$27,005.94
Wichita	93947.82	$713,424.24	88719.32	$798,496.11

To calculate the revenues and costs generated or incurred from curbside collection, we needed to determine the cost of recycling and sorting at large-scale MRFs. To calculate the net costs per ton of material in processed in a MRF, we used data from Resource Recycling Systems [31] to find operating, capital, and maintenance costs for MRFs of different tonnage capacity. A graphical representation of the processing and operating costs is shown below[31]:



Fargo, North Dakota
Single stream:

Using our model, we calculated that Fargo would generate 25,292.85 tons of recyclables annually using single stream curbside pickup. The operating cost is about $130 per ton for a dual stream MRF of the same tonnage capacity [31]. However, single stream MRFs have greater processing costs in the range of $10-15 per ton (averaged at $12.5), because of greater sorting required [4]. Using the revenue generated from selling recovered material, as calculated in the Drop-Off Center section to be $128.78, the net cost and total cost are:

Net cost per ton = ($130+ $12.5)- $128.78 + = $13.72 per ton

*Total cost = 25,292.85 tons * $1.22 per ton + Collection cost = $347,017.90 + $205,787.20 = $552,805.1*

Dual stream:

Using our model, we calculated that Fargo would generate 25,933.39 tons of recyclables annually using dual stream. The operating cost is about $130 per ton. Thus, net cost and total cost are:

Net cost per ton = $130 - 128.78 = $1.22 per ton
*Total cost = 25,933.39 tons * $1.22 per ton + Collection cost = $31,638.74 + $319,383.30 = $351,022.04*

Price, Utah
Single stream:

Using our model, we calculated that Price would generate 2,064.37 tons of recyclables annually using single stream. A mini MRF, with an annual tonnage of 2,649, is sufficient. The operating cost is about $245.62 per ton for single stream. Thus, net cost and total cost are:

Net cost per ton = $245.62 - 128.78 = $116.84

*Total cost = 2,064.37 tons * $116.84 per ton + Collection cost = $24,526.54 + $241,201 = $265,727.53*

Dual stream:

Using our model, we calculated that Price would generate 2,064.37 tons of recyclables annually using dual stream. A mini MRFis again sufficient. The operating cost is about $218.47 per ton for dual stream. Thus, net cost and total cost are:

Net cost per ton = $218.47 - 128.78=89.69

*Total cost = 2064.37 tons * $89.69 per ton + Collection cost = $24,526.54 + $27,005.94 = $212,159.29*

Wichita, Kansas

Single stream:

Using our model, we calculated that Wichita would generate 93,947.82 tons of recyclables annually using single stream. Thus, net cost and total cost are:

Net cost per ton = ($95 + $12.5) - $128.78 = -$21.28

*Total cost = 93,947.82 tons * -$21.28 per ton + Collection cost = -$3,173,557 + $713,424.24 = -$1,285,785.61*

As the cost is negative, the city receives a profit.

Dual stream:

Using our model, we calculated that Wichita would generate 88,719.32 tons of recyclables annually using dual stream. Thus, net cost and total cost are:

Net cost per ton = $95 - $128.78 = -$33.78

*Total cost = 93,947.82 tons * -$33.78 per ton + Collection cost = -$3,173,557.36 + $798,496.11 = -$2,375,061*

The city again receives a profit.

Pay-As-You-Throw

We can apply the Pay-As-You-Throw revenue formulas calculated in the Drop-Off Centers section:

Total revenue$_{PAYT}$ = *Revenue*$_{PAYT,\ Recycle}$ + *Revenue*$_{PAYT,\ Don't\ recycle}$ = *((4.5 lbs - 3.375 lbs) * Container price/pound - \$0.28/365) * Population*$_{Recycle}$ + *(4.5 lbs * Container price/pound - \$0.28/365) * Population*$_{Don't\ recycle}$

Given that 40% of people to whom curbside recycling is available recycle [16], we calculated the total revenue each city can expect from a Pay-As-You-Throw program alongside curbside recycling. The variable "x" is used to represent the container price per pound, which is up to the city to set.

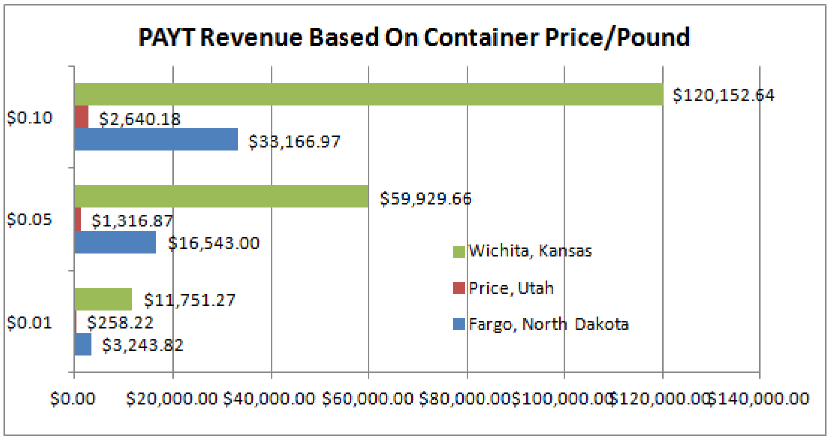
Fargo, North Dakota
Total revenue$_{PAYT}$ = *(1.125 lbs * x - \$0.28/365) * (.40 * 105,549 people*$_{Recycle}$*) + (4.5 lbs * x - \$0.28/365) * (105,549 - .40 * 105,549 people*$_{Don't\ recycle}$*)*

Price, Utah
Total revenue$_{PAYT}$ = *(1.125 lbs * x - \$0.28/365) * (0.40 * 8,402 people*$_{Recycle}$*) + (4.5 lbs * x - \$0.28/365) * (8,402 - 0.40 * 8,402 people*$_{Don't\ recycle}$*)*

Wichita, Kansas
Total revenue$_{PAYT}$ = *(1.125 lbs * x - \$0.28/365) * (0.40 * 382,368 people*$_{Recycle}$*) + (4.5 lbs * x - \$0.28/365) * (382,368 - 0.40 * 382,368 people*$_{Don't\ recycle}$*)*

The following table provides the total revenue generated by a PAYT program if the container price per pound were \$0.01, \$0.05, or \$0.10.

Container Price/Pound	Fargo, North Dakota	Price, Utah	Wichita, Kansas
$0.01	$3,243.82	$258.22	$11,751.27
$0.05	$16,543.00	$1,316.87	$59,929.66
$0.10	$33,166.97	$2,640.18	$120,152.64



3. Testing the Models

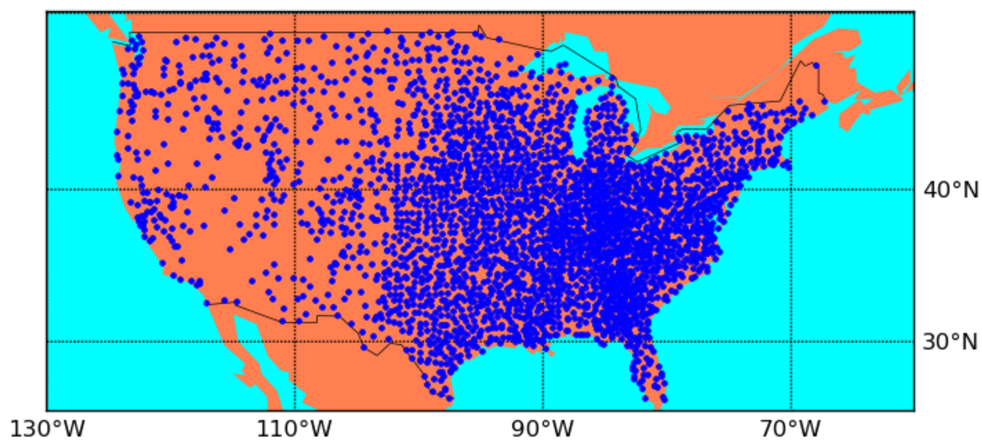
To test our models for accuracy, cities with a current drop-off recycling, single-stream curbside collection, or dual-stream curbside collection program can be run through the models. The population, area, and other required attributes of the city will be input into our models, and the accuracy of our models would be confirmed if the model results for annual amount recycled and net annual cost to the city are similar to the values in reality.

4. Recommendations

When designing a recycling program, the city should identify markets for recycled materials. The characteristics of the market determine how recyclables should be collected, processed, and eventually sold [32].

To extend our model to the national level, we took US Census data from 2000 which recorded the population density. We then tested each county. On the national level, the EPA should strongly encourage recycling programs for almost every county or region, particularly in denser, less rural regions.

The diagram below marks all the centers of all the counties where at least one of our three proposed recycling programs turns a profit to the community, based on the models were proposed earlier.



In general, across the U.S., very small cities such as Price, Utah will incur losses from a recycling program. A drop-off program cannot be used to full advantage because much of the potential coverage area of one drop-off center lies beyond the city limits. In relatively large, densely populated cities such as Wichita, Kansas, dual-stream curbside collection is generally recommended to bring the highest profits. This is due to low participation in drop-off recycling; on average, only 15.5% of potentially covered households participate. The revenue benefits of a pay-as-you-throw initiative must be balanced against the cost of its unpopularity among citizens. A pay-as-you-throw initiative is generally recommended for small cities such as Price, Utah that seek to adopt a recycling program but incur losses no matter what the program. In these cases, a pay-as-you-throw initiative is recommended to offset losses to the city.

III. CONCLUSION

Effective recycling programs are critical for cities to address the waste accumulation in landfills. Based on our models, we conclude that drop-off centers, curbside collection, and pay-as-you-throw initiatives can all be feasible recycling programs, depending on the population and area of a given city. All the models are resistant to minor changes in the input values and can be applied to any city.

The population growth of the U.S. has a notable effect on the change in the amount of plastic waste discarded in landfills each year. Partly because U.S. population growth has been linear in recent years, we determined that a linear model was most appropriate for predicting the amount of plastic waste discarded. Our linear model predicts that 35.1 million tons of plastic waste will be discarded in 2023, an increase of 13% over 2010.

Using a drop-off program, Fargo, North Dakota, and Wichita, Kansas would both generate profits from the sales of recovered materials. The net profits leave an unpopular PAYT initiative unnecessary. In Price, Utah, however, a drop-off program sustains losses because of the very small size of the city. For this reason, we recommend that Price adopt a PAYT initiative to raise revenues and offset costs of the drop-off program.

Using any curbside program, single-stream or dual-stream, Fargo, North Dakota and Price, Utah incur losses. In both cities, a drop-off program is recommended: in Fargo, because a drop-off generates profits and in Price, a drop-off generates less losses than a curbside collection. In Wichita, Kansas, however, both a single and dual stream curbside collection generate a profit, leaving all three programs feasible. Dual-stream curbside collection is strongly recommended, however, for the highest profits.

Nationally, small cities generally incur losses with any recycling program, as seen in our model results for Price, Utah. Dual-stream curbside collection is generally recommended for large, densely populated cities, who can take advantage of efficiencies of scale. The revenue benefits of a PAYT initiative must be balanced against the cost of its unpopularity among citizens, though it is recommended for small cities to help offset their losses. Recycling has environmental benefits for any city but is especially important for large, densely populated cities, where it has economic as well as environmental benefits.

BIBLIOGRAPHY

1. "2010 Facts and Figures Fact Sheet (PDF) - US Environmental ..." 2012. 4 Mar. 2013 <http://www.epa.gov/wastes/nonhaz/municipal/pubs/msw_2010_rev_factsheet.pdf>
2. 2013. <http://5gyres.org/what_is_the_issue/the_problem/>
3. "Municipal Solid Waste | Wastes | US EPA." 2004. 3 Mar. 2013 <http://www.epa.gov/msw/>
4. "Dual Stream" vs. "Single Stream" Recycling Programs | UMBC ..." 2012. 3 Mar. 2013 <http://umbcinsights.wordpress.com/2012/08/28/dual-stream-vs-single-stream-recycling-programs/>
5. "Curbside Collection Study - Eureka Recycling." 2010. 3 Mar. 2013 <http://www.eurekarecycling.org/page.cfm?ContentID=72>
6. "Organizing a Community Recycling Program." 2012. 3 Mar. 2013 <https://www.bae.ncsu.edu/topic/vermicomposting/pubs/ag473-11-community-recycle.html>
7. "Pay-As-You-Throw Programs| Conservation Tools | US EPA." 2003. 3 Mar. 2013 <http://www.epa.gov/payt/>
8. "Moving to Fargo, ND| Fargo, ND Moving Companies." 2009. 4 Mar. 2013 <http://www.upack.com/moving-companies/north-dakota/fargo/>
9. "Explore Utah - Getting Around in Utah - Navigating Utah's Streets ..." 2011. 4 Mar. 2013 <http://www.exploreutah.com/GettingAround/Navigating_Utahs_Streets.shtml>
10. Wichita travel guide - Wikitravel." 2005. 4 Mar. 2013 <http://wikitravel.org/en/Wichita>

11 "A Brief History of Plastic - The Brooklyn Rail." 2008. 4 Mar. 2013 <http://www.brooklynrail.org/2005/05/express/a-brief-history-of-plastic>
12 "National Intercensal Estimates (2000-2010) - U.S Census Bureau." 2011. 4 Mar. 2013 <http://www.census.gov/popest/data/intercensal/national/nat2010.html>
13 "Estimating Consumer Willingness to Supply and Willingness to Pay ..." 3 Mar. 2013 <http://le.uwpress.org/content/88/4/745.refs?related-urls=yes&legid=wple;88/4/745>
14 "Gasoline and Diesel Fuel Update - Energy Information ... - EIA." 2011. 3 Mar. 2013 <http://www.eia.gov/petroleum/gasdiesel/>
15 "Fuel Economy | National Highway Traffic Safety Administration ..." 2010. 4 Mar. 2013 <http://www.nhtsa.gov/fuel-economy>
16 "Ohio EPA 2004 Drop-Off Recycling Study." 2010. 3 Mar. 2013 <http://www.epa.ohio.gov/portals/34/document/general/swmd_drop_off_study_report.pdf>
17 "Municipal Solid Waste Generation, Recycling, and Disposal in - US ..." 2009. 3 Mar. 2013 <http://www.epa.gov/osw/nonhaz/municipal/pubs/msw2008rpt.pdf>
18 "Recycling Stats | GreenWaste Recovery." 2008. 3 Mar. 2013 <http://www.greenwaste.com/recycling-stats>
19 "Enterprise Portal Information - Home." 2006. 4 Mar. 2013 <http://www.portal.state.pa.us/>
20 "SCRAP TIRE RECYCLING - Tennessee." 2008. 4 Mar. 2013 <http://www.tn.gov/environment/swm/pdf/TFscraptires.pdf>
21 "Recycled Wood Products - Sonoma Compost." 2007. 4 Mar. 2013 <http://www.sonomacompost.com/recycle_wood.shtml>
22 "Plywood Thickness and Weights: Sanded Nominal Thickness 1/4 3 ..." 2008. 4 Mar. 2013 <http://parr.com/PDFs/PG_plywoodthickness.pdf>
23 "Recycling and Composting Online." 4 Mar. 2013 <http://www.recycle.cc/freepapr.htm>
24 "Glass." 2010. 4 Mar. 2013 <http://www.ndhealth.gov/wm/recycling/Glass.htm>
25 "Daily Scrap Metal Prices - Maxi Waste Limited." 2007. 4 Mar. 2013 <http://www.maxiwaste.co.uk/scrapdailyprices.php>
26 "Aluminum: How Sustainable is It? | Streamline." 2010. 4 Mar. 2013 <http://www.streamlinemr.com/articles/aluminum-how-sustainable-is-it>
27 "Textile Recycling FAQs - Trans-Americas Trading Co." 2011. 4 Mar. 2013 <http://tranclo.com/recycling-faq.asp>
28 "Solid Waste Publications for Sullivan County - Waste Management ..." 2013. 4 Mar. 2013 <http://waste.uvlsrpc.org/index.php/solid-waste-publications-sullivan-county/>
29 "Georgia Department of Community Affairs." 2007. 4 Mar. 2013 <http://www.dca.state.ga.us/main/quickmenuListing.asp?mnuitem=PROG>
30 Skumatz, LA. "Measuring Source Reduction: Pay-As-You-Throw Variable Rates as ..." 2000. <http://www.epa.gov/osw/conserve/tools/payt/pdf/sera.pdf>
31 "Extending Recycling's Reach with MRF Hub and ... - NC SWANA." 4 Mar. 2013 <http://www.ncswana.org/files/2012%20Fall%20Presentations/Frey-NC%20SWANA%20MRF%20Optimization%20presentation%20100212.pdf>
32 "Organizing a Community Recycling Program." 2012. 4 Mar. 2013 <https://www.bae.ncsu.edu/topic/vermicomposting/pubs/ag473-11-community-recycle.html>